Correlation Lengths of the Lieb-Liniger Model

Ovidiu I. Pâţu and Andreas Klümper

Institute for Space Sciences, Bucharest and Bergische Universität Wuppertal

Outline

1. The Lieb-Liniger model
2. Correlation lengths via the QTM method
3. The LL model as the continuum limit of the XXZ spin chain
4. The field-field correlator. Asymptotic behavior and numerical results
5. The density-density correlator. Asymptotic behavior and numerical results

The Lieb-Liniger Model

The Hamiltonian (Lieb-Liniger 1963)

$$H_{NLS}=\int_0^l dx\,\left[\partial_x\Psi^\dagger(x)\partial_x\Psi(x)+c\Psi^\dagger(x)\Psi^\dagger(x)\Psi(x)\Psi(x)-\mu\Psi^\dagger(x)\Psi(x)\right]\,,$$

$c>0$ coupling constant, μ chemical potential, l the length of the system ($\hbar=2m=1$) with m the mass of the particles.

$$[\Psi(x),\Psi^\dagger(x')]=\delta(x-x')\,,\quad [\Psi(x),\Psi(x')]=[\Psi^\dagger(x),\Psi^\dagger(x')]=0$$

$$H_{NLS}=-\sum_{i=1}^{n}\frac{\partial^2}{\partial x_i^2}+c\sum_{1\leq i<j\leq n}\delta(x_i-x_j)-\mu n$$

Bethe Anstaz equations

$$e^{ik_jl}=\prod_{s\neq j}^{n}\frac{k_j-k_s+ic}{k_j-k_s-ic}\,,\quad j=1,\cdots,n$$

Energy spectrum $\overline{E}(\{k\})=\sum_{j=1}^{n}\overline{e}_0(k_j)\,,\quad \overline{e}_0(k)=k^2-\mu\,,$

Thermodynamics and correlation functions

Grand-canonical potential per unit length (Yang and Yang 1969):

$$\phi(\mu, T) = -\frac{T}{2\pi} \int_{-\infty}^{+\infty} \log\left(1 + e^{-\overline{\varepsilon}(k)/T}\right) dk$$

Yang-Yang equation:

$$\overline{\varepsilon}(k) = k^2 - \mu - \frac{T}{2\pi} \int_{\mathbb{R}} \overline{K}(k - k') \log\left(1 + e^{-\overline{\varepsilon}(k')/T}\right) dk' \,.$$

$\overline{\theta}(k) = i \log\left(\frac{ic+k}{ic-k}\right)$, $\quad \lim_{k\to\pm\infty} \overline{\theta}(k) = \pm\pi$; $\quad \overline{K}(k - k') = \frac{d}{dk}\overline{\theta}(k - k') = \frac{2c}{(k-k')^2+c^2}$

Temperature dependent correlation functions

$$\langle \mathcal{O} \rangle_T = \frac{\sum \langle \Omega | \mathcal{O} | \Omega \rangle e^{-E/T}}{\sum e^{-E/T}} \,,$$

- Field-field correlation function: $\langle \Psi^\dagger(x)\Psi(0)\rangle_T$
- Density-density correlation function: $\langle j(x) j(0)\rangle_T$ with $j(x) = \Psi^\dagger(x)\Psi(x)$

Previous results

Impenetrable limit ($c \to \infty$)

- Girardeau (1960); Lenard (1964),(1966); Vaidya and Tracy (1979); Jimbo, Miwa, Môri, Sato (1980); Its, Izergin, Korepin, Slavnov,Varzugin (1989-1993); Gangardt (2004)

Finite coupling strength

- TLL/CFT: Haldane (1981); Bogoliubov, Izergin and Korepin (1986); Berkovich and Murthy (1988)
- ABA: Bogoliubov and Korepin (1984); Izergin and Korepin (1984); Kitanine, Kozlowski, Maillet, Slavnov and Terras (2009),(2012); Kozlowski, Maillet and Slavnov (2011), Kozlowski and Terras (2011), Kozlowski (2011)

Method

Integrable lattice models at $T > 0$: Quantum Transfer Matrix

- Largest eigenvalue of QTM $\rightarrow$ Free energy of the system $F = -k_B T \log \Lambda_0(0)$
- Next largest eigenvalues $\rightarrow$ Correlation lengths

Problem: The QTM does not exist for continuum models!

- Continuum limit of the XXZ spin chain $\rightarrow$ Lieb-Liniger model (Kulish, Sklyanin 1979)
- Yang's thermodynamics from the XXZ spin chain QTM result (Seel, Bhattacharyya, Göhmann, Klümper 2007)
- Multiple integral representation for the correlation functions (Seel, Bhattacharyya, Göhmann, Klümper 2007; Seel, Göhmann, Klümper 2008)

General strategy: Obtain asymptotic expansions for the correlation functions of the XXZ spin chain and take the continuum limit.

The XXZ spin chain

The Hamiltonian: $H(J,\Delta,h) = H^{(0)}(J,\Delta) - hS_z$

$$H^{(0)}(J,\Delta) = J\sum_{j=1}^{L}\left[\sigma_x^{(j)}\sigma_x^{(j+1)} + \sigma_y^{(j)}\sigma_y^{(j+1)} + \Delta(\sigma_z^{(j)}\sigma_z^{(j+1)} - 1)\right], \quad S_z = \frac{1}{2}\sum_{j=1}^{L}\sigma_z^{(j)}$$

$\Delta = \cos\eta$ with $0 < \eta < \pi(|\Delta| < 1)$; $h < h_c = 8J\cos^2(\eta/2)$

Bethe Ansatz equations: $\left(\frac{\sinh(\lambda_j - i\eta/2)}{\sinh(\lambda_j + i\eta/2)}\right)^L = \prod_{s\neq j}^{n} \frac{\sinh(\lambda_j - \lambda_s - i\eta)}{\sinh(\lambda_j - \lambda_s + i\eta)}, \qquad j = 1, \cdots, n$

Energy spectrum: $E(\{\lambda\}) = \sum_{j=1}^{n} e_0(\lambda_j) - h\frac{L}{2}, \quad e_0(\lambda) = \frac{2J\sinh^2(i\eta)}{\sinh(\lambda + i\eta/2)\sinh(\lambda - i\eta/2)} + h$

Bare momentum: $p_0(\lambda) = i\log\left(\frac{\sinh(i\eta/2 + \lambda)}{\sinh(i\eta/2 - \lambda)}\right)$,

Scattering phase and kernel:

$\theta(\lambda) = i\log\left(\frac{\sinh(i\eta + \lambda)}{\sinh(i\eta - \lambda)}\right), K(\lambda) = \theta'(\lambda) = \frac{\sin(2\eta)}{\sinh(\lambda + i\eta)\sinh(\lambda - i\eta)}$

Continuum limit

XXZ spin chain	One-dimensional Bose gas
lattice constant $\delta = \mathcal{O}(\epsilon^2)$	
number of lattice sites $L = \mathcal{O}(1/\epsilon^2)$	physical length $l = L\delta$
interaction strength $J = 1/2$	particle mass $m = 1/2$
magnetic field $h = \mathcal{O}(\epsilon^2)$	chemical potential $\mu = (\frac{\epsilon^2}{\delta^2} + \frac{\epsilon^4}{4\delta^2} - \frac{h}{\delta^2})$
anisotropy $\Delta = \cos\eta = \epsilon^2/2 - 1$	repulsion strength $c = \epsilon^2/\delta$
inverse temperature β	inverse temperature $\overline{\beta} = \beta\delta^2$

Seel, Bhattacharyya, Göhmann, Klümper (2007); O.I.P. and Klümper (2013)
Spectral parameter $\frac{\epsilon}{\delta}\lambda = k$; $\eta = \pi - \epsilon$

- BAE (XXZ spin chain) $\rightarrow$ BAE (Bose gas)
- One-particle momentum and two-particle scattering
  $p_0(\lambda) \rightarrow \delta k\,, \quad \theta(\lambda) \rightarrow -\overline{\theta}(k)\,, \quad K(\lambda) \rightarrow -\frac{\epsilon}{\delta}\overline{K}(k)\,,$
- One-particle energy $\beta e_0(\lambda) \rightarrow \overline{\beta}\overline{e}_0(k)$
- $Z_{XXZ}(h,\beta) \equiv \lim_{L\to\infty} \sum_{\{\lambda\}} e^{-\beta E(\{\lambda\})} \rightarrow Z_{NLS}(\mu,\beta) \equiv \lim_{l\to\infty} \sum_{\{k\}} e^{-\beta \overline{E}(\{k\})}\,.$

XXZ spin chain correlators at low-T and vanishing magnetic field $\rightarrow$ temperature dependent correlators in the Bose gas at all T

The XXZ spin chain QTM

XXZ spin chain R-matrix:

$$\mathrm{R}(\lambda,\mu)=\begin{pmatrix}1&0&0&0\\0&b(\lambda,\mu)&c(\lambda,\mu)&0\\0&c(\lambda,\mu)&b(\lambda,\mu)&0\\0&0&0&1\end{pmatrix}\quad \begin{array}{l} b(\lambda,\mu)=\frac{\sinh(\lambda-\mu)}{\sinh(\lambda-\mu+i\eta)}\\ c(\lambda,\mu)=\frac{\sinh(i\eta)}{\sinh(\lambda-\mu+i\eta)}\end{array}$$

L- operators:

$$\mathrm{L}_j(\lambda,-u')=\sum_{a,b,a_1,b_1=1}^{2}\mathrm{R}^{aa_1}_{b\,b_1}(\lambda,-u')e^{(0)}_{ab}e^{(j)}_{a_1b_1}\,,\qquad \tilde{\mathrm{L}}_j(u',\lambda)=\sum_{a,b,a_1,b_1=1}^{2}\mathrm{R}^{b_1\;a}_{a_1\;b}(u',\lambda)e^{(0)}_{ab}e^{(j)}_{a_1b_1}\,,$$

$u'=-2iJ\sin\eta\frac{\beta}{N}$; N is the Trotter number;
$e^{(0)}_{ab}=e_{ab}\otimes\mathbb{I}_2^{\otimes L}$ and $e^{(i)}_{ab}=\mathbb{I}_2\otimes\mathbb{I}_2^{\otimes(i-1)}\otimes e_{ab}\otimes\mathbb{I}_2^{\otimes(N-i)}$
Monodromy matrix:

$$\mathrm{T}^{QTM}(\lambda)=\mathrm{L}_N(\lambda,-u')\tilde{\mathrm{L}}_{N-1}(u',\lambda)\cdots\mathrm{L}_2(\lambda,-u')\tilde{\mathrm{L}}_1(u',\lambda)$$

The QTM:

$$\mathrm{t}^{QTM}(\lambda)=\mathrm{tr}_0\mathrm{T}^{QTM}(\lambda)$$

The QTM spectrum and correlation functions

The eigenvalues of the QTM:

$$\Lambda(\lambda) = b(u',\lambda)^{N/2} e^{\beta h/2} \prod_{j=1}^{p} \frac{\sinh(\lambda - \lambda_j - i\eta)}{\sinh(\lambda - \lambda_j)} + b(\lambda, -u')^{N/2} e^{-\beta h/2} \prod_{j=1}^{p} \frac{\sinh(\lambda - \lambda_j + i\eta)}{\sinh(\lambda - \lambda_j)}$$

Bethe ansatz equations: $\left(\frac{b(u',\lambda_j)}{b(\lambda_j,-u')}\right)^{N/2} = e^{-\beta h} \prod_{j\neq k}^{p} \frac{\sinh(\lambda_j - \lambda_k + i\eta)}{\sinh(\lambda_j - \lambda_k - i\eta)}, \quad j = 1, \cdots, p.$

Largest eigenvalue in the $N/2$ sector ($p = N/2$): $F = -k_B T \log \Lambda_0(0)$

Longitudinal correlation ($m \to \infty$)

$$\langle \sigma_z^{(1)} \sigma_z^{(m+1)} \rangle_T = const + \sum_{i \in N/2 \text{ sector}} A_i\, e^{-\frac{m}{\xi_i^{(d)}}}, \quad 1/\xi_i^{(d)} = \log(\Lambda_0(0)/\Lambda_i^{(ph)}(0))$$

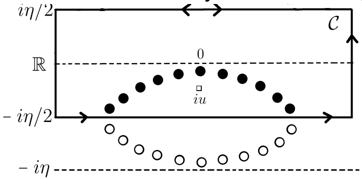
Transversal correlation ($m \to \infty$)

$$\langle \sigma_+^{(1)} \sigma_-^{(m+1)} \rangle_T = \sum_{i \in N/2-1 \text{ sector}} B_i\, e^{-\frac{m}{\xi_i^{(s)}}}, \quad 1/\xi_i^{(s)} = \log(\Lambda_0(0)/\Lambda_i^{(s)}(0))$$

Continuum limit $\langle \sigma_z^{(1)} \sigma_z^{(m+1)} \rangle_T \to \langle j(x) j(0) \rangle_T$; $\langle \sigma_+^{(1)} \sigma_-^{(m+1)} \rangle_T \to \langle \Psi^\dagger(x) \Psi(0) \rangle_T$

Bose gas thermodynamics from the XXZ QTM. I

NLIE for the auxiliary function, Integral expresion for the eigenvalue



$$\log \mathfrak{a}(\lambda) = -\beta h - \beta \frac{2J\sinh^2(i\eta)}{\sinh(\lambda + i\eta)\sinh\lambda} - \frac{1}{2\pi}\int_{\mathcal{C}} K(\lambda-\mu)\log(1+\mathfrak{a}(\mu))d\mu$$

$$\log\Lambda_0(0) = \frac{\beta h}{2} + \frac{1}{2\pi}\int_{\mathcal{C}} \frac{\sin\eta}{\sinh(\mu+i\eta)\sinh(\mu)}\log(1+\mathfrak{a}(\mu))d\mu\,.$$

At low-T neglect the upper part of the contour; $e^{-\varepsilon(\lambda)/T} = \mathfrak{a}(\lambda - i\eta/2)$, Klümper and Scheeren (2003)

$$\varepsilon(\lambda) = e_0(\lambda) + \frac{T}{2\pi}\int_{\mathbb{R}} K(\lambda-\mu)\log\left(1+e^{-\varepsilon(\mu)/T}\right)d\mu\,,$$

$$\log\Lambda_0(0) = \frac{h}{2T} + \frac{1}{2\pi}\int_{\mathbb{R}} p_0'(\lambda)\log\left(1+e^{-\varepsilon(\lambda)/T}\right)d\lambda$$

Bose gas thermodynamics from the XXZ QTM. II

XXZ spin chain (low T)

$$\log\Lambda_0(0)=\frac{h}{2T}+\frac{1}{2\pi}\int_{\mathbb{R}}p_0'(\lambda)\log\left(1+e^{-\varepsilon(\lambda)/T}\right)d\lambda$$

$$\varepsilon(\lambda)=e_0(\lambda)+\frac{T}{2\pi}\int_{\mathbb{R}}K(\lambda-\mu)\log\left(1+e^{-\varepsilon(\mu)/T}\right)d\mu$$

Using $\phi(\mu,\overline{T})=(f(h,T)+h/2)/\delta^3$ and $f(h,T)=-T\log\Lambda_0(0)$

Bose gas (all T)

$$\phi(\mu,T)=-\frac{T}{2\pi}\int_{-\infty}^{+\infty}\log\left(1+e^{-\overline{\varepsilon}(k)/T}\right)dk$$

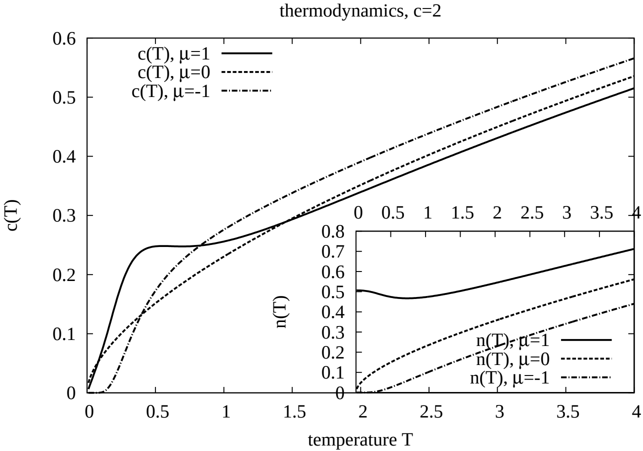
$$\overline{\varepsilon}(k)=k^2-\mu-\frac{T}{2\pi}\int_{\mathbb{R}}\overline{K}(k-k')\log\left(1+e^{-\overline{\varepsilon}(k')/T}\right)dk'\,.$$

Seel, Bhattacharyya, Göhmann, Klümper (2007)

Bose gas thermodynamics. Numerical results

Grand-canonical potential per unit length: $\phi(\mu, T) = -\frac{T}{2\pi}\int_{-\infty}^{+\infty} \log\left(1 + e^{-\overline{\varepsilon}(k)/T}\right) dk$

Yang-Yang equation: $\overline{\varepsilon}(k) = k^2 - \mu - \frac{T}{2\pi}\int_{\mathbb{R}} \overline{K}(k-k') \log\left(1 + e^{-\overline{\varepsilon}(k')/T}\right) dk'$.



Specific heat and particle density

- $\mu < 0$: gas phase $c(T), n(T) \simeq e^{-\frac{|\mu|}{T}}$
- $\mu = 0$: critical value $c(T), n(T) \simeq T^{1/2}$
- $\mu > 0$: CFT $c(T) \simeq T$, $n(T) \simeq const$.

Asymptotics of the field-field correlator

Asymptotic expansion (Klümper and OIP 2013)

$$\langle \Psi^\dagger(x)\Psi(0)\rangle_T = \sum_i \tilde{B}_i \, e^{-\frac{x}{\xi^{(s)}[\overline{v}_i]}}\,, \qquad x\to\infty$$

Correlation lengths

$$\frac{1}{\xi^{(s)}[\overline{v}_i]} = -\frac{1}{2\pi}\int_{\mathbb{R}} \log\left(\frac{1+e^{-\overline{v}_i(k)/T}}{1+e^{-\overline{\varepsilon}(k)/T}}\right) dk - ik_0 - i\sum_{j=1}^{r} k_j^+ + i\sum_{j=1}^{r} k_j^-$$

$$\overline{v}_i(k) = k^2 - \mu \pm i\pi T + iT\overline{\theta}(k-k_0) + iT\sum_{j=1}^{r}\overline{\theta}(k-k_j^+) - iT\sum_{j=1}^{r}\overline{\theta}(k-k_j^-) - \frac{T}{2\pi}\int_{\mathbb{R}} \overline{K}(k-k')\log\left(1+e^{-\overline{v}_i(k')/T}\right) dk'$$

$2r+1$ parameters: k_0 and $\{k_j^+\}_{j=1}^r$ ($\{k_j^-\}_{j=1}^r$) upper (lower) half of the complex plane; plus (minus) sign when k_0 is in the first (second) quadrant

$$1+e^{-\overline{v}_i(k_0)/T}=0\,,\quad 1+e^{-\overline{v}_i(k_j^\pm)/T}=0.$$

At low temperatures: k_0 can dive under the real axis; indented contour such that k_0 is above it

Field-field correlator $\langle\Psi^\dagger(x)\Psi(0)\rangle_T$: Leading term I

Correlation length

$$\frac{1}{\xi^{(s)}[\overline{v}]} = -\frac{1}{2\pi}\int_{\mathbb{R}} \log\left(\frac{1+e^{-\overline{v}(k)/T}}{1+\ e^{-\overline{\varepsilon}(k)/T}}\right) dk - ik_0$$

NLIE

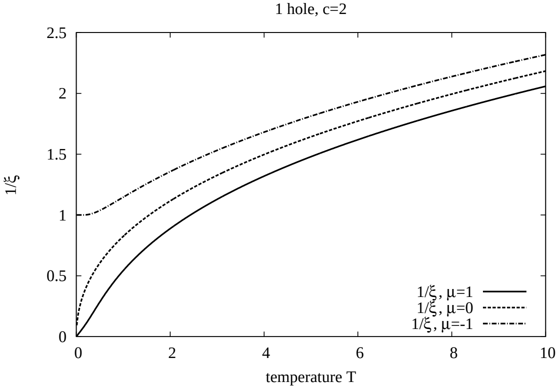
$$\overline{v}(k) = k^2 - \mu + i\pi T + iT\overline{\theta}(k-k_0) - \frac{T}{2\pi}\int_{\mathbb{R}} \overline{K}(k-k')\log\left(1+e^{-\overline{v}(k')/T}\right) dk'$$

- $\mu > 0$: k_0 is located in the upper half-plane at intermediate and high temperatures, at low-temperatures located in the lower half-plane
- $\mu < 0$: k_0 is imaginary

CFT ($\mu > 0$)

$$\langle\Psi^\dagger(x)\Psi(0)\rangle_T = \tilde{B}_0\left(\frac{\pi T/v_F}{\sinh(\pi Tx/v_F)}\right)^{\frac{1}{2\mathcal{Z}^2}} + \sum_{l=\pm1,\cdots}\tilde{B}_l\, e^{2ixlk_F}\left(\frac{\pi T/v_F}{\sinh(\pi Tx/v_F)}\right)^{\frac{1}{2\mathcal{Z}^2}+2l^2\mathcal{Z}^2}$$

Field-field correlator $\langle\Psi^\dagger(x)\Psi(0)\rangle_T$: Leading term II



- $\mu < 0$: gas phase $1/\xi(T)$ finite
- $\mu = 0$: critical value $1/\xi(T) \simeq T^{1/2}$
- $\mu > 0$: CFT $1/\xi(T) \simeq \frac{\pi T}{v_F}\frac{1}{2\mathcal{Z}^2}$

Impenetrable limit ($c \to \infty$): Its, Izergin and Korepin (1993)

$$\frac{1}{\xi} = \frac{1}{2\pi}\int_{\mathbb{R}} \log\left(\frac{e^{(k^2-\mu)/T}+1}{e^{(k^2-\mu)/T}-1}\right) dk + \sqrt{|\mu|}\,, \qquad \mu < 0$$

$$\frac{1}{\xi} = \frac{1}{2\pi}\int_{\mathbb{R}} \log\left|\frac{e^{(k^2-\mu)/T}+1}{e^{(k^2-\mu)/T}-1}\right| dk\,, \qquad \mu > 0$$

Asymptotics of the density-density correlator

Asymptotic expansion (Kozlowski, Maillet and Slavnov 2010)

$$\langle j(x)j(0)\rangle_T = const + \sum_i \tilde{A}_i\, e^{-\frac{x}{\xi^{(d)}[\overline{u}_i]}}\,, \qquad x\to\infty\,,$$

Correlation lengths

$$\frac{1}{\xi^{(d)}[\overline{u}_i]} = -\frac{1}{2\pi}\int_{\mathbb{R}} \log\left(\frac{1+e^{-\overline{u}_i(k)/T}}{1+\ e^{-\overline{\varepsilon}(k)/T}}\right) dk - i\sum_{j=1}^{r} k_j^+ + i\sum_{j=1}^{r} k_j^-\,,$$

$$\overline{u}_i(k) = k^2-\mu + iT\sum_{j=1}^{r}\overline{\theta}(k-k_j^+) - iT\sum_{j=1}^{r}\overline{\theta}(k-k_j^-) - \frac{T}{2\pi}\int_{\mathbb{R}}\overline{K}(k-k')\log\left(1+e^{-\overline{u}_i(k')/T}\right)dk'$$

The $2r$ parameters, $\{k_j^+\}_{j=1}^r$ ($\{k_j^-\}_{j=1}^r$) are located in the upper (lower) half of the complex plane and satisfy the constraint

$$1+e^{-\overline{u}_i(k_j^\pm)/T} = 0$$

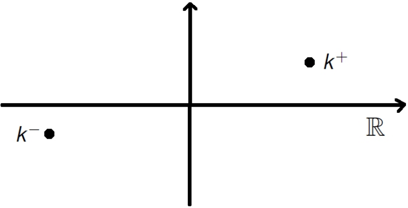
Density correlation function $\langle j(x)j(0)\rangle_T$: Next-leading term I

Correlation length

$$\frac{1}{\xi^{(d)}[\overline{u}]} = -\frac{1}{2\pi}\int_{\mathbb{R}} \log\left(\frac{1+e^{-\overline{u}(k)/T}}{1+e^{-\overline{\varepsilon}(k)/T}}\right) dk - ik^{+} + ik^{-} \,,$$

NLIE

$$\overline{u}(k) = k^2 - \mu + iT\overline{\theta}(k-k^{+}) - iT\overline{\theta}(k-k^{-}) - \frac{T}{2\pi}\int_{\mathbb{R}} \overline{K}(k-k')\log\left(1+e^{-\overline{u}(k')/T}\right) dk'$$

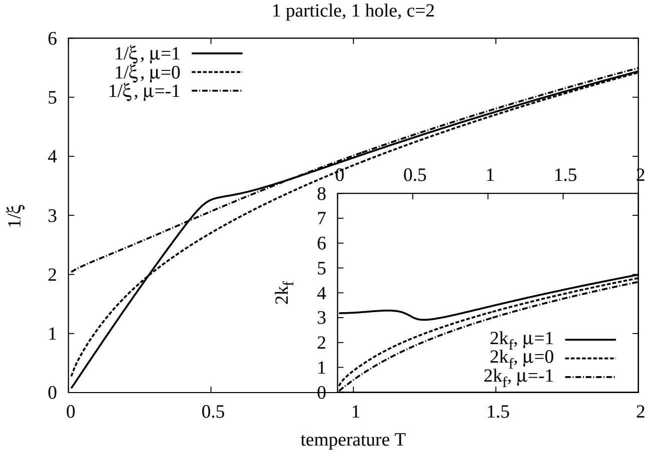


CFT expansion

$$\langle j(x)j(0)\rangle_T = \langle j(0)\rangle_T^2 + \frac{(T\overline{\mathcal{Z}}/v_F)^2}{2\sinh^2(\pi Tx/v_F)}$$

$$+\tilde{A}_1\, e^{2ixk_F}\left(\frac{\pi T/v_F}{\sinh(\pi Tx/v_F)}\right)^{2\overline{\mathcal{Z}}^2} + \cdots$$

Density correlation function $\langle j(x)j(0)\rangle_T$: Next-leading term II



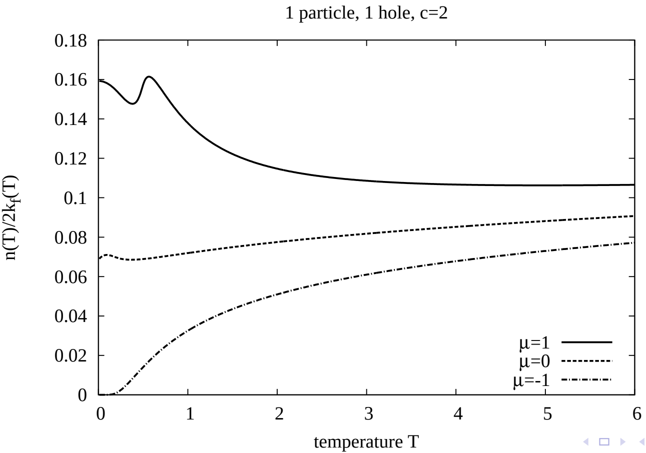
- $\mu < 0$: gas phase
  $1/\xi(T)$ finite
- $\mu = 0$: critical value
  $1/\xi(T) \simeq T^{1/2}$
- $\mu > 0$: CFT
  $1/\xi(T) \simeq \frac{2\pi T}{v_F}\overline{\mathcal{Z}}^2$
  $2k_F$ oscillation

Density correlation function $\langle j(x)j(0)\rangle_T$: Next-leading term III

Low-temperature

$$1/\xi(T) \simeq \frac{2\pi T}{v_F}\overline{\mathcal{Z}}^2\,, \qquad k_F(T) \simeq \pi n(T)$$

no smooth behavior of $n(T)/K_F(T)$



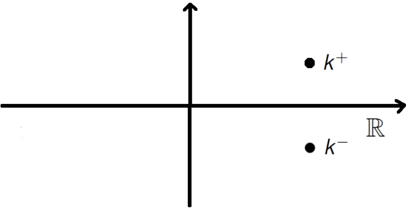
Density correlation function $\langle j(x)j(0)\rangle_T$: Leading Term I

Correlation length

$$\frac{1}{\xi^{(d)}[\overline{u}]} = -\frac{1}{2\pi}\int_{\mathbb{R}} \log\left(\frac{1+e^{-\overline{u}(k)/T}}{1+\,e^{-\overline{\varepsilon}(k)/T}}\right) dk - ik^{+} + ik^{-}\ ,$$

NLIE

$$\overline{u}(k) = k^2 - \mu + iT\overline{\theta}(k-k^{+}) - iT\overline{\theta}(k-k^{-}) - \frac{T}{2\pi}\int_{\mathbb{R}} \overline{K}(k-k')\log\left(1+e^{-\overline{u}(k')/T}\right) dk'$$

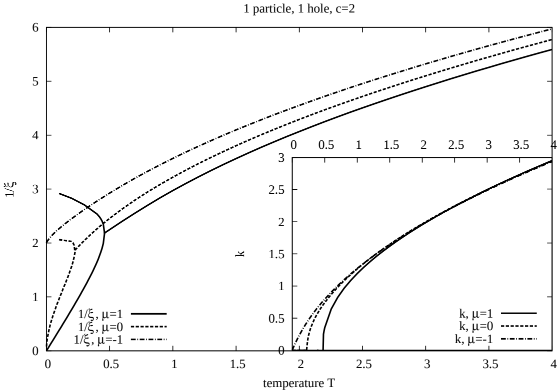


CFT expansion

$$\langle j(x)j(0)\rangle_T = \langle j(0)\rangle_T^2 + \frac{(T\overline{\mathcal{Z}}/v_F)^2}{2\sinh^2(\pi Tx/v_F)} + \sum_{l\in\mathbb{Z}^*} \tilde{A}_l\, e^{2ixlk_F}\left(\frac{\pi T/v_F}{\sinh(\pi Tx/v_F)}\right)^{2l^2\overline{\mathcal{Z}}^2}$$

Density correlation function $\langle j(x)j(0)\rangle_T$: Leading Term II

Leading term: crossover from non-oscillating to oscillating behavior at higher T



- $\mu > 0$: CFT $1/\xi(T) \simeq \frac{2\pi T}{v_F}$
- $\mu > 0$ and $\mu = 0$ crossover

- At low-T ($\mu \geq 0$) we have $k^+ = (k^-)^*$
- At high-T ($\mu \geq 0$) two solutions with the same correlation length and different $k^\pm$ which are not complex conjugate

Similar phenomenon: The XXZ spin chain

The XXZ spin chain (no magnetic field) $\cos \pi\eta = -\Delta \in (0,1)$ (Fabricius, Klümper and McCoy 1999)

$$H^{(0)}(\Delta) = \frac{1}{2}\sum_{j=1}^{L}\left[\sigma_x^{(j)}\sigma_x^{(j+1)} + \sigma_y^{(j)}\sigma_y^{(j+1)} + \Delta(\sigma_z^{(j)}\sigma_z^{(j+1)} - 1)\right]$$

At zero temperature

$$\langle\sigma_z^{(1)}\sigma_z^{(m+1)}\rangle_{T=0} \sim -\frac{1}{\pi^2\eta m^2} + (-1)^m\frac{C(\Delta)}{m^{1/\eta}}$$

At finite temperature

$$\langle\sigma_z^{(1)}\sigma_z^{(m+1)}\rangle_T = \sum_i A_i\left(\frac{\Lambda_i(0)}{\Lambda_0(0)}\right)^m$$

- $T < T_L(\Delta)$ leading $\Lambda_i(0)/\Lambda_0(0)$ real > 0, $A_i < 0$
- $T_L(\Delta) < T < T_U(\Delta)$ leading $\Lambda_i(0)/\Lambda_0(0)$ and A_i complex
- $T_L(\Delta) < T$ leading $\Lambda_i(0)/\Lambda_0(0)$ real > 0, $A_i > 0$

Future plans

Same method can be employed to obtain an efficient thermodynamic description of multi-component Bose and Fermi gases (Klümper and OIP, 2011)

- Correlation lengths for the 2 component Bose gas (partial results for the gas phase)
- Correlation lengths for the 2 component Fermi gas (attractive and repulsive)

THANK YOU !